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ABSTRACT

Intended as a resource to help mathematics teachers better understand how reading difficulties can affect student math performance, this annotated bibliography contains approximately 75 entries. The entries are arranged into the following categories: (1) reading skills needed in mathematics, (2) mathematical vocabulary and symbols, (3) comprehending mathematical materials, (4) problem solving, (5) mathematics and language, (6) readability of mathematical materials, and (7) other related topics. Additional sections of the bibliography provide a summary of research findings, conclusions based on those findings, and recommendations for classroom teachers. Appendixes offer suggestions and strategies for teaching the skills needed to read mathematics and games and activities to reinforce those skills. (FL)

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TEACHING READING IN THE MATHEMATICS CLASS

BY

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INTRODUCTION

In recent years, reading instruction in the content areas has been the subject of contention and acclaim among educators. There are various reasons for content area reading problems. Hollander (1977) maintained that most content teachers do not have the requisite skills to teach specialized reading techniques in their field. Clary (1974) added the level of student competence, the quality of content materials, and curriculum pressures as sources of content reading problems.

All of these difficulties are compounded in mathematics by the abstract nature of the subject (Bye 1975, Florida...1975, Smith and Kepner 1981, Texas...1979). Reading and mathematics have many common elements, but there are significant differences as well. "Mathematical material is generally characterized by its conciseness, abstractness, and complex relationships." (Collier and Redmond:805) Specific skills and techniques are required in reading mathematics, in addition to the general reading skills that elementary teachers teach their students to apply when reading basal narrative material. Since mathematics and reading form the heart of the elementary school curriculum, they must be taught in conjunction instead of as separate endeavors (Feeman 1973). Reading teachers must strive to supply children with general reading skills. However, the mathematics teacher must take the major responsibility for teaching students to read mathematics because of the highly abstract symbolic system and technical terminology (Florida...1975). Interpretation of mathematical symbols and vocabulary requires content knowledge.

Manzo (1975:2) contended that there are more math illiterates than people who cannot read. He cited the difficulties experienced by students in working with math concepts and word problems to be "unparalled among current educational dilemmas." Buxton (1980) reiterated this concern when she pointed out that 80-90% of the adult, British population dislikes mathematics. Schell (1982) claimed that math is the most difficult material to read, with more concepts per word, sentence, and paragraph than any other subject. Perhaps this contention contributes heavily to the causes of the dilemma. There are no "cure all" techniques, but it is possible to identify and analyze the factors that make math difficult to read and comprehend; and then to attempt to reduce these difficulties.

STATEMENT OF THE PROBLEM

The content area of mathematics is an abstract subject that presents unique difficulties to students of all ages. Some of these difficulties can be attributed to specific reading problems associated with mathematics. Effective reading of English does not imply the ability to read mathematical language. Many teachers neglect to teach the reading skills needed to accurately decode and comprehend mathematical materials.

PURPOSE OF THE STUDY

Information is needed that will enable teachers of mathematics to better understand how reading difficulties affect math performance. The purpose of this study was to explore the problems associated with reading mathematics, to make educators more aware of these problems, and to

discuss ways that children can be helped to overcome the obstacles. A list of practical techniques and activities was compiled to provide the content teacher with the confidence and specialized knowledge required to begin a program aimed toward teaching these essential skills.

ORGANIZATION OF THE STUDY

The study is an annotated bibliography of literature on teaching children to read mathematics. Subsections are used to help organize the material. However, there is some overlapping between areas since the problems are interrelated. The first section identifies the many skills and difficulties involved, and the authors' suggestions to educators. The following sections are divided according to the major area of difficulty being discussed: vocabulary and symbols, comprehending mathematical materials, solving word problems, the relationship of mathematics and language, readability of mathematical textbooks and materials, and other related topics.

LIMITATIONS OF THE STUDY

This is a review of recent literature on the topic, except for a few early studies that were consistently referred to by later researchers. The articles are a representative sampling of information available on the problems of reading mathematics. The study is not intended to be an exhaustive presentation of all aspects concerning each individual area of difficulty.

GLOSSARY

The following words are defined as dictated by their use in mathematics.

abstract -- expressing a quality apart from any concrete objects, theoretical.

arithmetic -- a branch of mathematics that deals with computation or problem solving using real numbers.

computation -- the use of mathematical symbols to determine the number which results from a given operation.

concrete experiences -- manipulation of tangible objects or experiences.

context clues -- while reading, get clues from pictures, diagrams, examples, and familiar words in the sentence, problem, or paragraph.

directionality -- the direction that the eyes must move when reading mathematical symbols (up, down, left, right, diagonally, and around).

math -- a shortened form of the word mathematics.

mathematics -- the study of numbers, form, arrangement, and associated relationships, using rigorously defined literal, numerical, and operational symbols (American Heritage 1976:806).

mathematical -- (the adjective form of the noun mathematics) relating to mathematics.

problem solving -- application of knowledge and skills to intellectual or physical problems -- real or simulated. Problem solving is the most sophisticated reading task in mathematics (Earle:7). Commonly referred to as word problems, story problems, and verbal problems.

symbols -- a character, abbreviation, or mark which represents an idea, concept expression, quantity, or operation (Florida...:1).
Examples: +, =, x, 5, Δ, { }

syntax -- the way, or order, in which words are put together to form phrases or sentences.

visual materials -- graphic materials such as charts, diagrams, graphs, number lines, and pictures.

vocabulary

general -- words or symbols found in everyday life (chair, love, $, closed).

technical -- terms peculiar to mathematics (quotient, polygon, ≠, algebra).

special -- everyday vocabulary which takes on different or specialized meaning in the context of mathematics (plane, point, odd, base, closed).

ANNOTATIONS

A Review of Reading Skills Needed in Mathematics

Anderson, Betty, and Joels, Rosie Webb, Primary Grade Content Reading: Why and How, Educational Resources Information Center, ED 209 633, November, 1980, 22 pp.

Anderson and Joels explored the features of primary science, social studies, literature, and mathematics materials that can be manipulated to improve content area reading instruction. They stated that acquisition of appropriate language is crucial to success in mathematics. Four basic types of math words must be taught: technical, specialized meanings, process signals, and general. Also, Anderson and Joels stressed that dealing effectively with symbols requires three specific competencies: (1) associating a word or phrase with the symbol, (2) expressing an idea in objects, pictures, and symbols, and (3) awareness that many symbols have multiple meanings. The authors suggested some creative activities to reinforce these skills. In solving word problems, the student must be able to find the main idea and the supporting details, organize these comprehended parts, and use them as they follow directions. They also offered activities and strategies for teaching students to read pictorial and graphic materials.

Barney, Leroy, "Problems Associated With the Reading of Arithmetic," The Arithmetic Teacher 19:131-133, February, 1972.

Barney explored some of the general reading problems of children that should be given specialized attention by the mathematics teacher. He described procedures that have proven helpful to children in learning technical vocabulary, words with multiple meanings, punctuation peculiar to mathematics, and abbreviations. Barney stated that difficulties with the readability of narrative problems is often related to word and sentence length. Rewriting of some problems may be necessary. Children need help in using interpretive and analytical reading skills to determine unnecessary or insufficient data. Since arithmetic text is usually exact, terse, precise, and humorless, Barney suggested that the students orally restate the problem in their own words.

Barney, Leroy, "The First and Third R's," Today's Education: NEA Journal 62:57-58, March, 1973.

Barney clarified that the writing in arithmetic texts can be classified into four types: algorism problems, narrative problems, narration and explanation, and illustrations for computations. The last three are more dependent upon reading skills. The child with reading difficulties is so busy trying to pronounce the words, that he doesn't have time to really interpret them and transfer them into meaningful data. Barney made a number of suggestions to make arithmetic reading easier. Crucial word problems (those vital to understanding a concept) could be rewritten in short phrases and outline form. Difficult vocabulary words should be taught prior to an assignment. A few problems with insufficient data should be given to see if students can read interpretively and analytically. A child can be taught to restate a problem in his own words. The teacher should provide experiential background with well-written trade books. The fast students may need supplementary materials, while slower students would benefit from manipulative materials.

Cassidy, Jack, and Sharkey, Verena, "'Rithmetic Reading," Teacher 94:50-2+, January, 1977.

Cassidy and Sharkey reported on Project C.A.R.E. in Newark, Delaware and presented some of the most successful activities that were included in their reading/math centers. The games and activities are designed to be used independently in stations. Five problem areas in reading mathematics were identified and practical activities for either remedial or developmental situations were explained. The five areas of need were word/symbol recognition, vocabulary, comprehension, problem-solving, and survival (functional literacy skills).

Collier, Calhoun C., and Redmond, Lois A., "Are You Teaching Kids to Read Mathematics?" Reading Teacher 27:804-809, May, 1974.

Collier and Redmond asserted that the reading ability of youngsters is clearly related to success; not only in story problem solving, but in all mathematics. Different skills and techniques are required to read mathematics than other curricular material. The arithmetic text differs from others in format and style. They emphasized that mathematical material is characterized by its conciseness, abstractness, and complex relationships. This requires an adaptation of reading rate and style. Different eye movements are often required. The authors stressed vocabulary development and the need to differentiate between social and mathematical usage for many words. They suggested that children keep a glossary. Collier and Redmond then discussed the skills needed for successful problem solving and some questions pupils should be encouraged to ask themselves when working word problems.

Dolgin, Ann B., "Improvement of Mathematical Learning Through Reading Instruction," The High School Journal 61:59-69, November, 1977.

Dolgin maintained that every teacher may incorporate reading skills without sacrificing content. She discussed vocabulary and comprehension skills and included some classroom activities. Dolgin stressed vocabulary development at three stages of understanding: literal, functional, and conceptual. Comprehension skills must also be taught at three levels: literal, inferential, and critical. A model lesson for comprehension was presented. Dolgin suggested the use of an informal diagnostic survey, developed by herself, to determine how well a reader may understand a textbook. It may also be used to determine the appropriateness of certain curriculum materials.

Earle, Richard A., Teaching Reading And Mathematics, An I.R.A. Service Bulletin, International Reading Association, Newark, Delaware, 1976, 88 pp.

Earle presented a model of mathematical reading based on Barrett's definition of reading. His model included the following hierarchial activities:

1. Perceiving symbols
2. Attaching literal meaning, first to vocabulary, then to explicit ideas.
3. Analyzing relationships
4. Solving word problems

This stresses that word problems are the most sophisticated reading task in mathematics. Earle presented strategies for teaching each level of his model, and then suggested many practical examples for implementing the strategies. Also included were six appendices helpful to the classroom teacher: constructing a tachistoscope, words appearing most frequently in math books, a structured overview, high interest materials, reading-study skills, and a cloze test and key.

Earp, N. Wesley, Reading In Mathematics, Educational Resources Information Center, ED 036 397, May 2, 1969, 24 pp.

Earp noted that there is a broad basis of research pointing to definite reading skills important to mathematics achievement. Earp stressed that the readability levels of texts must be lowered, teacher training for teaching content reading needs to be improved, and in-service programs for practicing teachers should be implemented. Earp characterized verbal arithmetic material as being conceptually packed with a high density factor which requires at least three kinds of adjustment: varied eye movements, a slower reading rate, and intentional rereading. A student's arithmetic vocabulary includes technical words, which do not greatly parallel or overlap reading text vocabulary, as well as symbols and signs. The student must be taught to read for the total idea and the sequence and relationship of ideas.

Earp, N. Wesley, "Procedures for Teaching Reading in Mathematics," Arithmetic Teacher 17:575-579, November, 1970.

Earp asserted that immediate action on reading in the content areas is needed in elementary and secondary classrooms. He suggested two forms of action: college teacher training in content area reading must be improved, and in-service programs for practicing teachers must be initiated. Earp listed suggestions, agreed upon by researchers, for dealing with the teaching of reading in mathematics. Arithmetic material is highly conceptualized, which requires three adjustments: slower reading rate, varied eye movements, and rereading. The child is usually expected to read mathematics for several purposes; such as, grasp total idea, note sequence, relate two ideas, and find a key question. Children need to be taught to read graphs, charts, and diagrams. Earp recommended that technical vocabulary be consistently taught, using the inductive method when possible. He then outlined the reading procedure for word problem solving that is widely prescribed by authorities.

Florida State Department of Education. Reading the Language of Mathematics, Educational Resources Information Center, ED 134 972, 1975, 105 pp.

This booklet was published to acquaint mathematics teachers and reading teachers with the skills needed to read the language of mathematics and to provide sample activities to help develop these skills. The authors stated that since mathematics is an abstract science involving the use of a system with highly specialized symbolism and technical vocabulary, it is suggested that the mathematics teacher accept major responsibility for teaching students to read mathematics. Topics explored are: vocabulary and symbols, comprehension, study techniques, and voluntary mathematical reading. This is an excellent resource booklet for teachers.

Hater, Mary Ann; Kane, Robert B.; and Byrne, Mary Ann, "Building Reading Skills in the Mathematics Class," The Arithmetic Teacher 21:662-668, December, 1974.

Hater, Kane, and Byrne stated that children do not ordinarily know how to read mathematics books. The need for acquiring the basic skills of reading mathematics has become more crucial with increased interest in individual and small group instruction. The authors identified thirteen skills that are used in reading the language of mathematics. These skills were briefly explained and illustrated with examples. The authors stressed that none of the skills stand alone; each is dependent on others. The best place to learn how to read mathematics is in the mathematics class.

Henrichs, Margaret, and Sisson, Tom, "Mathematics and the Reading Process: A Practical Application of Theory," Mathematics Teacher 73:253-257, April, 1980.

This article reported on a reading program initiated in a junior high school mathematics department. The program was based on the belief that reading is an ongoing process at all levels, and must be integrated with course content. Skills stressed in the program were reading mathematics textbooks, learning vocabulary and symbols, comprehension skills, listening skills, and adjustment of reading rate. The authors explained how directed reading lessons can be used for understanding word problems. The last aspect of the program showed how students were required to master the basic addition, subtraction, multiplication, and division facts using real-life situations and the newspaper.

Kane, Robert B.; Byrne, Mary Ann; and Hater, Mary Ann, Helping Children Read Mathematics, American Book Co., New York, 1974, 150 pp.

Kane, Byrne, and Hater first described the language of mathematics and compared it with ordinary English. The authors next reported on the development of readability formulas for mathematical materials. To accomplish this, a list of familiar mathematics vocabulary was established and the cloze technique was adapted for use with mathematical writing. Thirteen special skills dealing with the reading of mathematics were then discussed, along with hints and some activities for teaching them. The skills necessary for perception and comprehension when reading the language of mathematics were divided into the following categories: (1) reading speed, (2) ordering symbols, (3) reading with paper and pencil, (4) reading graphic materials, (5) page organization, (6) main ideas and important details, (7) words with many meanings, (8) using mathematical words, (9) seeing and understanding symbols, (10) object, idea, words, symbols, (11) problem solving, (12) computing, and (13) searching for meaning using context clues.

Krulik, Stephen, "To Read or Not to Read, That is the Question!" Mathematics Teacher 73:248-252, April, 1980.

Krulik stated that mathematical problem solving and reading comprehension are highly correlated. Math teachers are the best qualified to teach reading in mathematics. Krulik pointed out that the written language of mathematics is constantly encountered--on the chalkboard, in the textbook, and even on directions and activity cards during concrete experiences. Activities were suggested for some basic reading skills: not all mathematics is read left to right, math should be read in a slow, step-by-step procedure, some words have different meanings in math than everyday conversation. Use of the cloze procedure to improve comprehension of the mathematical textbook was advocated by Krulik.

Mathematics Resource Project, University of Oregon, Didactics and Mathematics. Creative Publications, Inc., Palo Alto, CA, 1978, 187 pp.

The chapter in this book on reading in mathematics presented some recommendations from various experts in this field:

1. Consult language arts teachers and reading specialists for help.
2. Diagnose student reading abilities.
3. Teach students how to use the textbook and its features.
4. Make mathematical vocabulary and symbols meaningful.
5. Provide many opportunities for students to read mathematics.
6. Give explicit attention to the reading of charts, diagrams, tables, graphs, and measuring tools.
7. Make students aware that reading mathematics requires a much slower reading rate.
8. Encourage recreational reading of books with mathematical themes.
9. Give the very poor reader encouragement for even slight gains, and try tape recording material for him to follow.
10. Suggestions were then given for helping students read word problems.

Phillips, Jo McKeeby, "Reading Mathematical Content," The Instructor, March, 1968, pp. 120, 136, 138.

Phillips stated that elementary teachers should teach reading and mathematics all day long. Practice and instruction should be given in reading appropriate content materials. She offered activities and suggestions to meet the following needs when children read mathematical material:

1. Give specific motivations for reading. (children should read to answer questions)
2. Help children integrate explanatory material with displays. (the child should use both hands)
3. Teach children to read graphic materials and give them the opportunity to make their own.
4. Make technical vocabulary (symbols and words) meaningful. (Phillips advocated the hear-say-see-read-write sequence)
5. Teach special skills for reading story problems. (she suggested the language experience approach)

Smith, Cyrus F., Jr., and Kepner, Henry S., Jr., Reading in the Mathematics Classroom, Educational Resources Information Center, ED 207 836, National Education Association, Washington, D.C., 1981, 64 pp.

In this monograph the authors identified several of the major problems students have in reading mathematics. Smith and Kepner proposed an instructional framework for a mathematics lesson. They maintained that the abstract material of mathematics is more difficult to relate to one's cognitive framework. They noted that the structured overview has been shown to assist teacher and student in learning abstract material, and presented the strategy and an example of it. Smith and Kepner advocated the development of mathematics vocabulary and listed some strategies to accomplish this. The authors cited research that showed difficulties with word problems are more likely caused by students' inability to synthesize information than by reading difficulty. They also reviewed the use of the cloze procedure to improve comprehension in mathematics, and discussed the issue of using readability formulas with mathematics textbooks.

Texas Education Agency, Learning Partners: Reading and Mathematics, Educational Resources Information Center, ED 179 422, 1979, 45 pp.

This booklet was published for both reading specialists and mathematics teachers. It described many of the skills needed by students to read mathematics effectively and provided sample activities that may be used to develop these skills. One of the topics discussed was reading mathematical words and symbols with emphasis on directionality. The following areas of comprehension were examined: understanding main ideas and details, following directions, interpreting graphic materials, reading mathematical sentences, and reading and solving word problems. Important study techniques were then identified, which included adjustment of reading rate, textbook organization, and SQ3R. This booklet appeared to be based on the same outline as the publication by the Florida State Department of Education in 1975.

Thomas, Ellen Lamar, and Robinson, H. Alan, Improving Reading in Every Class: A Sourcebook for Teachers, Allyn and Bacon, Inc., Boston, 1977, 590 pp.

Three major suggestions for teaching reading in mathematics were offered by Thomas and Robinson in this book. First, some student guidesheets were prepared on the techniques of reading the "new math", mastering vocabulary, and a higher-level reading approach for mathematics. Next, the authors advocated the use of various procedures to diagnose weaknesses, teach mathematical vocabulary, help students use the textbook effectively, give more interesting assignments, and promote mathematical thinking outside the classroom. Finally, Thomas and Robinson examined some possible ways of improving problem solving abilities through improving reading.

Treacy, John P., "The Relationship of Reading Skills to the Ability To Solve Arithmetic Problems," Journal of Educational Research 38:86-95, October, 1944.

Treacy conducted this study in 1940 which is still being cited in some articles today. It was one of the first studies to determine which specific reading skills were related to the ability to solve arithmetic problems. A comparison of good and poor achievers in problem solving in arithmetic on 15 reading skills resulted in these findings: Good achievers were found to be better than poor achievers in the following reading skills:

1. arithmetic vocabulary
2. vocabulary (isolated words)
3. retention of stated details
4. drawing inferences from context
5. integration of dispersed ideas
6. perceiving relationships
7. quantitative relationships
8. vocabulary in context
9. reading level

No significant differences were found in the following skills:

1. predicting outcomes
2. understanding directions
3. rate of comprehension
4. general information
5. grasp of central thought
6. interpretation of content

Wagner, Jennifer, "Learning to Read Math," Curriculum Review 16:364-365, December, 1977.

Wagner stated that reading difficulties in math can be divided into two major categories: expression and comprehension. She pointed out that successful comprehension depends on overcoming problems in expression. Wagner maintained that students should be encouraged to give an oral interpretation of mathematical symbols, phrases, and procedures, since memory and learning are enhanced by verbalization. She cited two major difficulties in translating math material from a written to an oral form: lack of knowledge of the symbol system, and an inability to sequence the symbols correctly. Memorization of symbols is necessary since reading word attack skills cannot be utilized. There is not a one-to-one correspondence between symbols and spoken words (2 can mean two, twenty, second, etc.). Sequencing symbols often requires back-and-forth, up-and-down, or diagonal eye motion. The standard reading skills of a primary child are not sufficient to even read 516. Specific mathematical reading skills must be taught.

Mathematical Vocabulary and Symbols

Buxton, Laurie, "Symbol Inhibition," Mathematics Teaching 90:17, March, 1980.

Buxton, who is a member of the Inner London Education Authority, contended that 80-90% of the adult population profess a distaste for mathematics. The nature of mathematical symbols and the notations we form from them are great inhibitors to learning. There are two levels of barriers to understanding symbols: cognitive and affective. In the cognitive sense, unfamiliarity with symbols prevents a student from having any schema to which to attach the problem in order to make a start. Affectively, anxiety in mathematics is easily induced, which seriously impedes progress. Buxton maintained that mystery and fear are attached to symbols. She proposed that reasons for why people cannot learn mathematics be sought in the realm of feelings instead of rationality and logic.

Ciani, Alfred J., "Mastering Word and Symbol Language in Mathematics," School Science and Mathematics 81:371-377, May/June, 1981.

Ciani agreed with other educators that mathematics is comprised of two languages: mathematical symbols and word symbols. Competence in both is imperative. Ciani discussed the reading process as a hierarchy of skills: word recognition, literal understanding, interpretation of meaning, and application of meaning. He stressed that before meaning or understanding can take place, the reader must be able to first recognize and pronounce the language and mathematical symbols. Ciani stated that this can be accomplished through activities in the areas of sight word exercises, context clues, structural analysis, phonic clues, and dictionary skills. He gave several suggestions to help teachers teach the first step of perceiving and pronouncing mathematical language and symbols.

Georgia State Department of Education, Reading Mathematics, Educational Resources Information Center, ED 105 407, Atlanta. Offices of Instructional Services, 1975, 38 pp.

This booklet discussed two processes involved in reading mathematics: eye movements and relating the appropriate word or symbol to the corresponding mathematical idea. Seven different kinds of eye movements were identified and examples given of each. Many kinds of words can be found in mathematics; some are primarily mathematical words, some from everyday speech that take on new meanings, and some that have several different mathematical meanings. The authors pointed out that preconceived meanings of words often need to be overcome before mathematical meanings can be substituted. Many examples of each kind of words were offered along with suggestions for teaching them. Reading and mathematics teachers were urged to cooperatively explore techniques used in reading that may successfully be adapted to mathematics.

Kulm, Gerald, "Sources of Reading Difficulty in Elementary Algebra Textbooks," Mathematics Teacher 66:649-652, November, 1973.

Kulm conducted a study to determine the variables significant in the reading difficulty of elementary algebra textbooks. He identified two types of written material: explanatory and illustrative. Kulm's study produced several relevant findings for mathematics teachers. The study revealed that vocabulary is important, but the symbolism of mathematics was far more significant in making algebra difficult for elementary students to read. Kulm proposed that the difficulty students have in writing equations from word problems may be attributed to their inexperience with symbolism rather than reading difficulty. The importance of simple translation tasks is probably underestimated by teachers, according to Kulm. Extra practice should be given in reading or writing the narrative form of symbolic equations.

Pachtman, Andrew B., and Riley, James D., "Teaching the Vocabulary of Mathematics Through Interaction, Exposure, and Structure," Journal of Reading 22:240-244, December, 1978.

Pachtman and Riley stressed that if the student does not understand the vocabulary of word problems, it will be difficult to solve the problem, and also to understand the underlying concepts. Students are expected to cope with four types of terminology: technical vocabulary, symbols, everyday vocabulary used in a mathematical context, and general vocabulary. The authors recommended the structured overview because it provides students with the opportunity to incorporate new vocabulary and concepts into their existing cognitive framework. In small groups, students can discuss and contemplate concepts represented by vocabulary and discern relationships among these concepts. Pachtman and Riley enumerated and summarized nine steps in the structured overview technique, and utilized some exemplary problems to further explain the steps involved in the strategy.

Schell, Vicki J., "Learning Partners: Reading and Mathematics," Reading Teacher 35:544-548, February, 1982.

Schell stated that reading math involves abstractions, specialized symbolism, and technical terminology. She claimed that research indicates that math is the most difficult material to read. It has more concepts per word, per sentence, and per paragraph than any other subject. Schell examined the specialized vocabulary in terms of conceptualizing definitions, using a math dictionary, using affixes and root words, and learning specialized or multiple meanings. She then offered suggestions for reading symbols and graphic materials. The fact that directionality and spatial relationships often cause difficulty in reading mathematics was stressed. Schell noted that children need practice in reading and writing mathematical sentences in as many forms as possible. She then identified students' difficulty of translating word problems into symbols and presented some guidelines for teaching this skill.

Skrypa, Aldona, Effects of Mathematical Vocabulary Training on Problem Solving Ability of Third and Fourth-Graders, Educational Resources Information Center, ED 172 169, April, 1979, 92 pp.

Skrypa conducted a study to determine whether systematic mathematical vocabulary instruction would have any effect on the problem solving ability of third and fourth-graders. The experimental group was taught 120 of the most frequently appearing words in mathematics texts, using filmstrips and movies, concrete models, and pictures. Drill and reinforcement were provided through discussions and games, many of which were included in appendices. The experimental group made statistically significant gains on the experimenter-designed Mathematical Vocabulary Test and the Stanford Diagnostic Mathematics Test. Skrypa concluded that there is a significant relationship between the knowledge of specific mathematical terms and the solution of problems involving these terms. His appendices included a list of the vocabulary and symbols taught, a list of audio-visual materials, sample game illustrations, and the Mathematics Vocabulary Test that he designed (forms A and B).

Sullivan, Kathryn, Vocabulary Instruction in Mathematics: Do the 'Little' Words Count?, Educational Resources Information Center, ED 210 637, December, 1981, 9 pp.

A study was conducted to determine if instruction in the "little" words used in mathematics would improve children's performance on word problems. Thirty-eight intermediate students participated in the study. The control group was drilled in basic addition, subtraction, multiplication, and division facts for three weeks, while the experimental group received instruction in the meanings of 50 vocabulary words, such as "the", "is", "ten", "many", "first", etc. Comparison of pretest-posttest scores indicated that the students as a whole made significant gains, however, no significant differences were found. Drill on basic facts was no more effective than vocabulary instruction in improving computation of word problems. Sullivan included the list of 50 "little" vocabulary words and their meanings.

Taschow, Horst G., "Reading in Mathematics," Improving College and University Teaching XX:312-314, Autumn, 1972.

Taschow discussed some of the difficulties inherent in the reading of mathematics. Effective reading in the language of mathematics requires understanding of arithmetic numerals, literal numbers, operational symbols, and expressions of relationships. Concrete experiences with operational symbols are needed, rather than memorization of definitions, principles, and generalizations. Taschow emphasized the importance of learning technical vocabulary and specialized meanings for general words. In working to overcome mathematical reading inefficiencies, Taschow advocated a Group Informal Reading Inventory, a visual-auditory-kinesthetic-tactile approach, and a five-step guide to more effective reading.

Comprehending Mathematical Materials

Bye, M.P., Reading in Mathematics and Cognitive Development, Educational Resources Information Center, ED 124 926, 1975, 18 pp.

Bye contended that mathematics is an abstract subject. Its content is often presented in symbolic and generalized form and a very specific language. Bye acknowledged that there are many reasons for students' inability to read mathematics. However, after a brief review of Piaget's model of cognitive development, Bye concluded that many seemingly easy mathematical terms assume cognitive processes that may not yet be available to the student. Some preliminary research has supported this analysis. To overcome the resultant reading problems in mathematics, Bye proposed that students must be provided with broader experiences. These experiences will build greater cognitive development in the conceptual area, so that words can acquire more specific and deeper meanings.

Carney, John J., Organizing Behaviors that Facilitate Comprehension in Content Materials, Educational Resources Information Center, ED 163 401, March, 1978, 15 pp.

Carney stated that content teachers can aid comprehension skills by presenting reading materials in different ways that allow the student to internalize the subject matter. Carney discussed the use of using context to teach the word meanings of technical vocabulary. He also explained the use of the structured overview to introduce new vocabulary and concepts. Carney illustrated a technique for teaching the process of drawing inferences by using a visual model. He then described a strategy called embedded organizers, that can be used to arouse reader's interest and focus their attention to improve critical reading skills. Finally, he explained that helping students see the structure of content can facilitate learning.

Feeman, George F., "Reading and Mathematics," The Arithmetic Teacher 20: 523-529, November, 1973.

Feeman reported on a project, supported by the International Reading Association, to bring mathematics teachers and reading teachers together to share ideas and assist each other. First Feeman discussed similarities and differences between the teaching of reading as a skill and the teaching of reading in mathematics. Next he provided some practical exercises as examples of how to teach mathematical symbols and vocabulary more effectively. Last Feeman presented three study-guides (on problem solving, absolute value, and open sets) as an alternate strategy. The author maintained the importance of having a variety of strategies including lecture, discussions, indirect inquiry, direct inquiry, methods of individualization, programmed approaches, use of multimedia, etc. He claimed that the properly constructed study guide can be a powerful instrument that combines content and process and has capabilities for diagnosis, review, guided study, motivation, and enrichment.

Hater, Mary Ann, and Kane, Robert B., The Cloze Procedure as a Measure of the Reading Comprehensibility and Difficulty of Mathematical English, Educational Resources Information Center, ED 040 881, Purdue University, Lafayette, Indiana, 1970, 25 pp.

Hater and Kane are active and well-known researchers on the topic of reading in mathematics. In this study they proposed to validate the cloze procedure as a measure of the comprehensibility of mathematical English. The results supported the hypothesis that cloze tests over mathematical English passages are highly reliable measures and valid predictors of the reading comprehensibility of mathematical English passages for grades 7-12.

Hollander, Sheila K., Reading the Special Language of Mathematics, Educational Resources Information Center, ED 137 726, May, 1977, 17 pp.

Hollander pointed out that reading of mathematical material requires specific reading skills such as noting details and weighing their importance, following directions, organizing facts, making inferences, judging relevant and irrelevant information, and seeing relationships. Teachers cannot assume that students can transfer the skills used to read a basal reader to technical math terminology and symbols. Instruction in the special oral language of math should begin in kindergarten. For the students to understand the abstract language of math, manipulation of concrete objects must take place first.

Lees, Fred, "Mathematics and Reading," Journal of Reading 19:621-626, May, 1976.

Lees stressed that mathematics teachers at all levels can improve their students' mathematical learning by improving their ability to read mathematical material. Lees defined four basic patterns of writing in mathematical texts: short paragraph setting of a problem situation, explanatory paragraphs, graphs and charts, and numerous symbols, signs, and formulas. Mathematics teachers can use reading techniques and materials to help students learn mathematics by diagnosing their needs, constructing and using comprehension aids, and assigning homework in a realistic fashion. Diagnosis can be accomplished through readability scales, the cloze test, an informal reading inventory, and standardized tests. Eight suggestions for comprehension aids were offered by Lees: informal reading inventory, cloze procedure, study guides, reading guides, vocabulary study, symbol interpretation, adjustment of reading rate, and the PQ4R study technique. He also noted some guidelines for productive assignments.

Maffei, Anthony C., "Instructional Objectives, Comprehension, and Mathematics," Reading Improvement 10:37-39, Fall, 1973.

Because of the concise and unique vocabulary and symbolism used to explain mathematics, Maffei claimed that students of all ages will encounter reading comprehension problems when told to "evaluate, reduce, solve, simplify," etc. Maffei advocated the use of instructional objectives (I.O.s) along with the usual directions to insure better comprehension of mathematics. The I.O.s should specify, in behavioral terms, what the learner will be doing when carrying out the directions. Maffei advised that the objectives can also include time limits and expected performance level, if desired. I.O.s may prove too helpful for above average students, but for the poorer student, more emphasis is placed on his ability to apply concepts than to recall them. Several examples of I.O.s were given by Maffei, such as the following:

Problem	Usual Directions	I.O.
$(x+3)(x-8)$	Find the product.	Multiply each factor of two terms by using the distributive rule of multiplication over addition.

O'Mara, Deborah A., "The Process of Reading Mathematics," Journal of Reading 25:22-30, October, 1981.

O"Mara critically appraised a number of theoretical and research studies and reported her conclusions. A cross-classification of three reading factors (reading ability, specific reading skills, and readability issues) with five mathematical language contexts (pictorial, computational, verbal, text, and test) was used to highlight literature review. Most of the studies noted inconsistent results. O"Mara emmerated ten major conclusions that emerged from the studies and offered some promising teaching suggestions. She concluded that theoretical studies of all five mathematical language contexts report that reading comprehension is important for success in math. O"Mara emphasized the need to carefully and critically examine the findings of research.

Rayburn, Elaine, An Exploration of the Cloze Procedure in Arithmetic Reading, Educational Resources Information Center, ED 102 530, March, 1974, 9 pp.

Rayburn conducted this study to determine if a child's mathematical reading ability could more easily and readily be determined by the use of the cloze procedure. She constructed two cloze mathematical reading tests using passages from sixth and seventh grade mathematics texts. The results showed that most of the students taking the sixth grade level test were at their independent or instructional levels, while nearly all of the students taking the seventh grade level test were at their instructional or frustrational reading levels. A statistically significant correlation was found between the cloze test and the students' IQ scores. Rayburn advocated the use of the cloze test by mathematical teachers as an aid in grouping purposes.

Riley, James D., and Pachtman, Andrew B., "Reading Mathematical Word Problems: Telling Them What To Do Is Not Telling Them How to Do It," Journal of Reading 21:531-534, March, 1978.

Riley and Pachtman asserted that specific guidance is necessary for the student to overcome the difficulties of reading and understanding word problems. The language of word problems is compact, with "hidden" concepts and relationships. The authors adapted a reading guide for use with word problems. The guide helps the student comprehend the problem on three levels: (1) literal level -- facts of the problem; (2) interpretive level -- underlying concepts or ideas; and (3) applied level -- depicting the problem numerically. The guide presents possible alternatives for the student to choose from. Riley and Pachtman claimed that reading guides are more helpful for some students than the Directed Reading Method.

Thelen, Judith N., "Just Because Kids Can't Read Doesn't Mean They Can't Learn! or Every Science and Mathematics Teacher Is Not A Teacher of Reading," School Science and Mathematics 97:457-464, October, 1979.

Thelen pointed out that science and mathematics teachers agree that their primary job is to help students master the content of the subjects they teach. If the textbook is excessively relied on, the poorer readers become frustrated. Thelen observed that teachers need a technique to help students master the concepts. Proper instructional sequence must be provided to help students relate new concepts to prior experience and knowledge. Thelen advocated the strategy of graphic organizers or the structured overview. These strategies evolved from the theories of Ausubel. Thelen explained the construction and use of graphic organizers, which consist of an arrangement of words or labels relevant to the important concepts in the lesson. Thelen also purported that the textbook should be easier to read when the student has sufficient background of the new material.

Problem Solving

Balow, Irving H., "Reading and Computation Ability as Determinants of Problem Solving," The Arithmetic Teacher 11:18-22, January, 1964.

Balow cited earlier studies that were unclear as to relationship between reading ability and arithmetic achievement. He felt that the results of some studies were limited by the uncontrolled effects of intelligence. Balow conducted a study to determine if both the level of general reading ability and the level of computational skill are significantly associated with problem solving ability. He secured all levels of reading ability and computation skill, while keeping IQ controlled. The results of Balow's study showed that general reading ability and computation ability both have a significant effect on problem-solving ability. With the effects of IQ controlled, above grade level scores on the reasoning test (problem solving) appear to bear a closer relationship to computation ability than to reading ability.

Cloer, Thomas, Jr., Factors Affecting Comprehension of Math Word Problems--A Review of the Research, Educational Resources Information Center, ED 209 655, December, 1981, 17 pp.

Cloer viewed the research on comprehension of word problems from three different perspectives: the math educator's view, the reading educator's view, and a pedagogical perspective. Math teachers concentrate on teaching math concepts, procedure, generalizations, logical exchanges, and number facts. Reading teachers stress linguistic and psycholinguistic comprehension theories. The characteristics of the learner are employed, seeking interaction of the text and reader. Both of these approaches are theory in search of data. The pedagogical perspective is data in search of theory. The behavior of the math student is directly modified rather than testing theories. Cloer maintained that all three perspectives have valid support and can aid educators in teaching the skills needed for solving word problems.

Cohen, S. Alan, and Stover, Georgia, "Effects of Teaching Sixth-Grade Students to Modify Format Variables of Math Word Problems," Reading Research Quarterly 16:175-200, 1981.

Cohen and Stover conducted three experiments with average achieving sixth-grade students. Three structural format variables of math word problems were identified as interfering with comprehension as a result of the first experiment. In the second experiment, word problems with both easier and difficult formats were given. Large differences in successful problem solution were noted in favor of the easy formats. In the last experiment, students were taught to modify the interfering formats to increase comprehension. The three specific format variables taught were: (1) to insert diagrams of the problem situation, (2) to reorder number sequences appropriately, and (3) to extract extraneous information. The results showed significant differences of unusual magnitude. The authors concluded that direct instruction in how to change format to make word problems more comprehensible greatly enhances the problem-solving performance of students.

Davidson, James E., "The Language Experience Approach to Story Problems," Arithmetic Teacher 25:28, 1977.

Davidson claimed that the language experience approach to teaching children to solve story problems eliminates some of the reasons for failure, namely: low reading comprehension, lack of experience with the problem situation, and irrelevancy of the problem. Davidson outlined the procedure and gave an example of its use. An integral part of the approach requires the student to write a story problem from an experience. Davidson asserted that story problem solving is definitely enhanced by story problem writing, since the student must include all of the needed information. The author contended that the following excellent teaching methods are employed in the language experience approach: the child is guided by the teacher through an activity rather than being a passive learner, and success is built in since the child is familiar with the problem situation and the language.

Dunlap, William P., and McKnight, Martha Brown, "Vocabulary Translations for Conceptualizing Math Word Problems," The Reading Teacher 32: 183-189, November, 1978.

Dunlap and McKnight identified the two major problems affecting children's ability to solve mathematical word problems as the three-level translation of vocabulary, and the conceptualization of the problem. The translation process among general, technical, and symbolic vocabularies and the thinking process within each vocabulary are essential to the conceptualization of the message contained in the problem. Most children need to be guided through the development of appropriate mental images and concepts for association with particular expressions in each vocabulary. The teacher must also help children interact with the writer of the word problem. The authors listed, explained, and illustrated eight steps in the reading/thinking process as it applies to mathematics.

Harvin, Virginia R., and Gilchrist, Mary A., Mathematics Teacher--A Reading Teacher? Educational Resources Information Center, ED 041 702, Indiana University, South Bend, School of Education, 1970, 11 pp.

Harvin and Gilchrist's review of research indicated a need for further study of the relationship between reading and problem solving ability. Two contrasting populations of third graders were given reading and arithmetic tests. The correlation coefficients were computed after changing achievement scores to percentile ranks. Results showed a positive relationship between problem solving and reading, however the relationship was not significant enough to be an accurate predictor of performance of one variable from the other. The authors stated that the need for teaching reading in mathematics was indicated. Special vocabulary and certain comprehension skills should be developed in the context of mathematics. Factors which need further study were suggested by the authors.

Hollander, Sheila K., "A Review of Research Related to the Solution of Verbal Arithmetic Problems," School Science and Mathematics 78: 57-70, January, 1978.

Hollander reviewed pertinent studies of student groups (grades 3-8) conducted between 1922 and 1969 to determine factors related to the solution of word (verbal) problems. The research studies reviewed were divided into seven categories of experimental emphasis: problem analysis, computation, general reading ability, specific reading skills, specificity of the problem statement, the problem situation, and language factors. Many factors have been considered pertinent to achievement in solving word problems, but Hollander concluded that no single one seemed to be indispensable. She stated that beyond recognition of the fact that intelligence, arithmetic skills, and reading skills are elements involved in successful problem solving, conclusions of most studies were neither in agreement nor were they consistent. Hollander indicated some areas for further profitable study.

Knifong, J. Dan, and Holtan, Boyd D., "A Search for Reading Difficulties Among Erred Word Problems," Journal for Research in Mathematics Education 7:227-231, May, 1977.

Knifong and Holtan pointed out that educators' assumption that reading plays a role in word problem difficulties persists, despite weak support. The authors acknowledged that reading is necessary for success in word problems, but the issue for their study was whether poor reading skills typically contribute to failure, or is failure due to some other factor(s). Children were asked questions about erred word problems to determine their comprehension of the problem, and were also asked to read the problems aloud. It was determined that poor reading ability accounted for less than 10% of these erred problems. The authors concluded that if reading affects word problem success, it was not revealed by the methods of this study. Some limitations of the study were given. Knifong and Holtan gave two recommendations to teachers: (1) help students develop computation skills, and (2) do not expect work on reading skills to correct word problem difficulties.

Linville, William J., "Syntax, Vocabulary, and the Verbal Arithmetic Problem," School Science and Mathematics 76:152-158, 1976.

Linville stated that research findings have shown that reading competence plays a vital role in the ability of elementary students to solve verbal problems. However, the specific kinds of reading skills needed have not been agreed upon. Linville conducted a study to determine whether the degree of syntax and/or the level of vocabulary used in word problems contribute significantly to difficulty of the problems when computational operations are held constant. Linville concluded that syntax and vocabulary both determine the difficulty of verbal arithmetic problems. Vocabulary level could be more crucial in determining success than syntax. A sub-hypothesis also showed that pupils with higher reading achievement scores can be expected to experience greater success in solving verbal arithmetic problems than pupils with low reading achievement.

Maffei, Anthony C., "Reading Analysis in Mathematics," Journal of Reading 16:546-549, April, 1973.

Maffei contended that the teaching of word problems is basically an implementation of reading study skills rather than mathematical skills. He advocated the application of PQ4R (a reading/study attack system) to word problems. He explained the six steps of the process: preview, question, read, reflect, rewrite, and review. Maffei proposed that PQ4R should prove to be an effective technique with average to below average students.

Stover, Georgia, "Rewording Arithmetic Word Problems For Sixth and Eighth Graders to Minimize Reading Difficulties," Reading Improvement 17:26-35, Spring, 1980.

Stover investigated the effect of format and reading variables upon the process of arithmetic word problem solving. She asked sixth and eighth grade gifted students to rewrite a random sample of typical textbook word problems. Stover then interviewed each student to determine their reasons for any structural or format changes and analyzed all changes made. There were twelve identified variables in 90 word problem rewrites:

1. simplified vocabulary
2. made a long sentence a series of shorter ones
3. removed extraneous information
4. made question sentence separate
5. added information or changed story line
6. changed order of numerical information
7. changed verb to present tense
8. changed noun/pronoun to proper noun
9. took out "about"
10. took out "if"
11. added diagram
12. gave clue

Suydam, Marilyn N., and Weaver, Fred J., "Research on Problem Solving: Implications for Elementary School Classrooms," Arithmetic Teacher 25:40-42, November, 1977.

Suydam and Weaver cited problem solving as an ultimate goal of mathematics instruction. The authors briefly summarized the main findings of research on problem solving. The following points were related to the reading of mathematics: activities stressing certain reading skills such as finding main ideas, making inferences, constructing sequences, and following directions may improve problem solving achievement. Many students need specific instruction on quantitative vocabulary. A common problem is failure to read to note details. Some problems should be presented in separate sentences instead of paragraph format. To aid poor readers, present problems orally or use a tape recorder. Emphasis on isolated word cues can be misleading, directing attention away from crucial relationships. Reading seems to be a secondary cause of difficulty, while computational difficulties are the major deterrent to correct answers.

Zweng, Marilyn J., "The Problem of Solving Story Problems," Arithmetic Teacher 27:2-3, September, 1979.

Zweng cited research studies that indicate lack of reading ability and computational skill are not the major factors contributing to poor problem-solving performance. The author maintained that the results of the 1972 NAEP Mathematics Assessment showed that deciding which operation (addition, subtraction, multiplication, or division) to perform is the major stumbling block to successful problem solving. Zweng observed that textbooks give children little guidance on how to select the appropriate operation, and that children have little language with which to communicate about problem solving. Teachers must learn to ask children questions that are operation related, such as "When you put together bunches of the same size, should you multiply or divide?" Zweng also suggested that texts need more story problems, with ample opportunity to solve one-step problems before asking children to solve complex problems.

Mathematics and Language

Aiken, Lewis R., Jr., "Language Factors in Learning Mathematics," Review of Education Research 42:359-385, Summer, 1972.

Aiken reviewed studies from the last fifty years pertaining to the effects of language factors on learning mathematics. He suggested some implications and ideas for further research. Aiken found correlations between reading ability and mathematics achievement in a representative sample of studies on this topic. He concluded that further research is needed to determine the effects of specific reading skills. General intelligence seems to be an unmeasured variable in many of the studies. From several studies, Aiken concluded that vocabulary level and syntactic structure are important variables in solving verbal arithmetic problems. The readability level of mathematics texts is frequently higher than the performance level of the readers. Aiken stated that it can be concluded that instruction in reading in general, or reading of mathematics in particular improves performance in the mathematics. Findings also stressed the importance of a particular language factor, that of verbal reading ability. Verbalization of concepts is strongly recommended, if compatible with learner abilities.

Boraks, Nancy; Rezba, Richard J.; and Van de Walle, John A., "Millions, Trillions, Zillions," Teacher 96:94-101, November, 1978.

The authors observed that children's natural curiosity about numbers and size involves language as they seek words and syntax to describe their world and imagination. Boraks, Rezba, and Van de Walle pointed out that there are specific ways to link language arts and mathematics. They presented 20 activities for grades 1-6 which included a math exercise followed by a related language experience. The goal is to provide children with language skills and math communication tools to describe their thoughts and experiences in words. The suggested activities were creative, and appear to be easy to follow and implement.

Brunner, Regina Baron, "Reading Mathematical Exposition," Educational Research 18:208-213, June, 1976.

Brunner stated that reading mathematical exposition requires certain abilities due to differences between mathematical and natural languages. Mathematical language is more precise, non-redundant (therefore easy to overlook a key word or phrase), and relatively unambiguous. Brunner identified some characteristics that a reader of mathematics should have: he must understand the use of variables, accommodate new terminology and notation, deal with the hierarchial development of definitions and the sequential organization of mathematics, and be prepared to work while he is reading. Brunner emmerated eight abilities needed by the learner to read mathematical exposition with comprehension.

Capps, Lelen R., "Teaching Mathematical Concepts Using Language Arts Analogies," The Arithmetic Teacher 17:329-331, April, 1970.

Capps claimed that there are many opportunities to use language arts examples to help explain mathematical concepts through the use of analogy. He presented specific examples that teachers might use when teaching some basic mathematical concepts. Once an analogy has been shown, the teacher should challenge students to find others. Capp reported that his experience with the technique has led to the conclusions that students enjoy the analogies, and an increase in understanding of concepts results in both mathematics and language arts. Some examples of the technique follow:

commutative property	He ate ice cream and cake.
$4+7=7+4$	He ate cake and ice cream.
associative property	Tom and Fred joined Mary.
$(3+4)+5=3+(4+5)$	Tom joined Fred and Mary.

Cooper, Frayda F., "Math as a Second Language," Instructor, October, 1971, pp. 76-77.

Cooper maintained that when children enter first grade they have had several years of oral language experience, but their oral mathematics language is very limited. She stressed the necessity of learning to speak mathematical language before a child can read or write it intelligently. Cooper stated that "just as reading is talk written down, an equation is mathematical talk written down." (p. 76) Oral mathematical language should be developed through experiences with manipulative and concrete materials, pictures and number lines, and constant verbal descriptions of what they are doing. Cooper further defined this philosophy by describing activities and procedures for teaching various math concepts by stressing the language of mathematics. She stressed that the reading of mathematics should be a natural product of language development.

Earp, N. Wesley, and Tanner, Fred W., "Mathematics and Language," Arithmetic Teacher 28:32-34, December, 1980.

Earp and Tanner stated that there are matters other than the sheer ability to decode and comprehend printed matter that relate to reading in mathematics. They reviewed studies that indicated that success in mathematics is effected by students' level of language sophistication. Earp and Tanner suggested that much more oral language and discussion of mathematical ideas be used in mathematics classes. The authors conducted a study of two sixth-grade classes and their math textbook. The text contained 519 basal and common words, and 197 mathematical words. The students could adequately decode both kinds of words, but comprehension of common words was twice as accurate as mathematical words. The use of context was very helpful in comprehending unknown words. Earp and Tanner recommended systematic instruction of mathematical vocabulary, rewriting or paraphrasing of mathematical statements, cloze procedure, language experience approach, and specific procedures for "talking" through word problems.

Gupta, I.D., "Mathematics is a Language: A Rationale for Mathematics Instruction," International Journal of Mathematical Education in Science and Technology 11:29-32, January-March, 1980.

Since mathematics is often referred to as a language, Gupta explored the implied meanings of such a statement. He first looked at the semiotic nature of mathematics, or how signs are created and transformed for meaning. Gupta discussed the three basic functions of mathematics education and then some implications for teaching strategy when viewing mathematics as a language. Mathematical symbolism can be analysed into the following categories:

1. Contentive symbols	
Numerals (nouns)	0,1,2,3,-10,1/2
Pro-numerals (pronouns)	a,b,x,AB
2. Operative symbols (prepositions)	+,-,x,log,(),d,$\sqrt{}$,$\int$,$\cup$,Σ
3. Relation symbols (verbs)	=,$\neq$,>,$\subset$,$\rightarrow$,$\perp$
4. Auxiliary symbols (auxiliaries)	:(so that), $\forall$(for all)

Specifically structured experiences are needed to learn mathematical symbolism.

Manzo, Anthony V., The Math Student/The Math Teacher/The Math Problem, Educational Resources Information Center, ED 114 767, 1975, 10 pp.

Manzo considered the difficulties students experience with mathematical concepts and word problems to be an unparalleled educational dilemma. There is not opportunity for incidental learning in the student's non-math world where daily living and math seem unrelated. Students are defective in their ability to reason abstractly, deficient in subskills, disrupted by conditions or anxiety, or incompatible with math because they have different cognitive styles. Manzo stated that math teachers tend to be less verbal and social. The notations and language of math are complex and terse, actually comparable to learning a new language. Manzo offered some steps toward the solution of these problems. The R/Q procedure was outlined in which the teacher serves as a model for questioning, language, and thinking behavior.

McCabe, Patrick P., The Effect Upon Comprehension of Mathematical Material Repatterned on the Basis of Oral Language, Educational Resources Information Center, ED 140 275, May, 1977, 21 pp.

McCabe maintained that the language structures used in natural discourse by students are better understood and should be used by math teachers whenever possible in written and oral exposition. McCabe conducted a study to examine the effect upon comprehension (as measured by cloze scores) of mathematical material that was rewritten using elementary and junior high students' language patterns. Specific syntactic structures were identified that seemed to inhibit or facilitate comprehension. The rewritten passage was then given to a second of two matched groups. The results suggested that rewriting material had the greatest effect on comprehension of expository portions as opposed to narrative parts of the passages. The author discussed implications of his study for classroom teachers.

Munro, John, "Language Abilities and Maths Performance," Reading Teacher 32:900-915, May, 1979.

Munro contended that low achievement in mathematics may be, in part, related to the use of inadequate skills in various areas of language performance. The child needs opportunity to comprehend and use math concepts and relations within his own language structure, instead of just mathematical symbols and grammar. Munro examined the difficulties children encounter in decoding math relationships expressed in ordinary language. Among these problems were conflicting vocabulary meanings, syntactic differences between math and ordinary language, verbal math statements containing multiple math operations, and math problems coded in normal language. Munro then discussed four types of errors made by children in reading and writing math statements that result, in part, from their inability to process spatial cues. Some learning experiences were presented to remediate these errors.

Phillips, Jo, "The Problem With Math May Be Reading," Instructor 89: 67-69, November, 1979.

Phillips concluded that math difficulties may often be difficulties with language. She listed and discussed eight items, linguistic in nature, that are basic to understanding mathematics:

1. Children need a solid knowledge of referents for number symbols and other math tokens.
2. Children need to understand the structure of symbols and words for numbers greater than ten.
3. "Little" words (articles, prepositions, conjunctions) must be interpreted correctly.
4. Common words have different interpretations.
5. Reading mathematics involves many different eye movement directions.
6. Teachers need to find out the exact message children are getting.
7. Children must know the basic facts on command.
8. Estimation is essential to reading comprehension whenever numbers are involved.

Sullivan, Kathryn, The Comprehension of Mathematical Language, A Communication Approach, Educational Resources Information Center, ED 197 287, December, 1980, 15 pp.

Sullivan applied a communication approach to mathematics. Symbolic sentences in math contain references to time, space, amount, and quality (relation references). These relation references can be used to find the matter references (number symbols). Sullivan maintained that the concepts of matter and relation words can help children perceive symbolic mathematical sentences as a form of communication in which the reader locates matter references (2, .8, 3X, etc.) using relation references (=, >, +, ⊥, etc.).

Readability of Mathematical Materials

Elliot, Peggy Gordon, and Wiles, Clyde A., "The Print is Part of the Problem," School Science and Mathematics 80:37-42, January, 1980.

Elliot and Wiles maintained that recent studies have shown the textbook to be the primary source of instructional support for most math teachers. They claimed that for many students, the textbook is just another stumbling block because of poor reading skills. Math teachers often are not able to teach the necessary reading skills. The authors stated that teaching and learning, dependent on print that cannot be processed, comes to a halt. Elliot and Wiles contacted publishers of five major secondary math series for information regarding the readability of their texts. They found that reading consultants are not utilized, and inadequate readability formulas for math materials are used. In an effort to determine the extent of the print problem, a Cloze test (as advocated by Hater and Kane) was given to 91 certified mathematics teachers. Over one-fourth were unable to deal easily with this textbook print being marketed and widely used at the eighth grade level. Some solutions were offered to this dilemma.

Heddens, James W., and Smith Kenneth J., "The Readability of Elementary Mathematics Books," The Arithmetic Teacher, November, 1964, pp. 466-468.

The purpose of this study by Heddens and Smith was to determine the readability level of five different commercial textbooks for elementary mathematics. The Spache readability formula was applied to the primary level books, and the Dale-Chall formula was applied to analyze the readability of the intermediate textbooks. (these formulas were identified as inappropriate for mathematical textbooks by Kane in 1970) Heddens and Smith maintained that arithmetic text should be written at a lower reading level than the grade in which it is to be used. They concluded from their study that the readability level of the selected textbooks was generally above grade level. There was also variation of reading level within the texts, and the easier reading material was not necessarily at the beginning. The authors reiterated the fact that successful reading material in arithmetic must be at the learner's independent reading level.

Kane, Robert B., "The Readability of Mathematics Textbooks Revisited," The Mathematics Teacher 5:296-298, November, 1970.

Kane, a prominent author on the topic of reading mathematics, stated that mathematics textbooks are written in more than one language. Mathematics authors attempt to communicate some of their most crucial ideas by means of symbol systems other than ordinary English. Kane contended that ordinary English and Mathematical English are sufficiently different that they require different skills and knowledge from readers to achieve reading comprehension. Kane cited numerous studies that applied readability formulas that were devised for ordinary English to mathematical textbooks. Kane asserted that these formulas (Spache, Flesch, and Dale-Chall) are inappropriate for rating mathematical textbooks for two reasons: (1) large segments of crucial reading material must be ignored because it is presented in symbolic systems, (2) the English sentences often contain words and phrases from the jargon of mathematics. The author pointed out that steps have been taken by himself and others to develop a readability formula for mathematics.

Smith, Edwin H.; Guice, Billy M.; and Cheek, Martha C., "Informal Reading Inventories for the Content Areas: Science and Mathematics," Elementary English 49:659-666, May, 1972.

The authors pointed out two major problems that face content teachers: determining the readability level of materials assigned to students, and determining the students' reading level in the particular content area. In this article Smith, Guice, and Cheek dealt with the second problem. They explained some characteristics of informal reading inventories such as their ability to note decoding and processing difficulties and to determine the independent, instructional, and frustrational reading levels of the student. The authors stressed that content reading materials should be on the student's independent or instructional level. A student can comprehend and remember what he reads more efficiently if he can read with less difficulty. Informal reading inventories for science and mathematics were presented, containing passages with readability levels of 2.5 to 6.5.

Related Topics

Cassidy, Jack, "Reading C.A.R.E.," Teacher 94:70-72, January, 1977.

Cassidy reported on Project C.A.R.E. (Content Area Reading Enrichment), adopted by Newark School District, Delaware. Its purpose was to identify and integrate reading skills particular to specific curriculum areas. Cassidy explained that kits were made for each subject containing games and independent activities designed to fulfill learning objectives. He then gave general descriptions of the learning goals for math, science, social studies, and English. Examples of sample activities for each kit were outlined. Cassidy stated that word problems in math are probably the most difficult for students and the most frustrating for teachers. Consequently, many of the math activities are designed to help students understand word problems.

Clary, Linda Mixon, The Why and a Little How -- Teaching Reading in the Content Areas, Educational Resources Information Center, ED 176 231, 1974, 7 pp.

Clary maintained that teachers must understand the reasons why students have trouble reading content materials before help can be provided. She discussed four sources of the content area reading problems: (1) student competence; (2) content materials; (3) curriculum pressures; and (4) teacher education. Clary prescribed the use of two reading approaches that can be modified for content area reading: the language experience approach and the individualized reading approach. A short description of each followed.

Gilmary, Sister, "Transfer Effects of Reading Remediation to Arithmetic Computation When Intelligence Is Controlled and All Other School Factors Are Eliminated," The Arithmetic Teacher 14:17-20, January, 1967.

Sister Gilmary based this study on the belief that the accepted principles underlying the understanding and use of language symbols also apply to arithmetic symbols. Gilmary stated that the purpose of her study was to determine if remedial instruction in both reading and arithmetic would produce greater gains in arithmetic computation than remedial measures in arithmetic only. Statistical analysis of the results revealed differences significant at the 5 percent level. Those children who received help in both reading and arithmetic showed significantly greater grade score gains in arithmetic computation than did those who received instruction in arithmetic only. Gilmary concluded that it would seem the teaching-learning effects of the reading classes had significant transfer value for arithmetic classes. Some limitations associated with the study suggested further research should be conducted.

Holladay, Joanne, and McGinty, Denise, Can a Reading Teacher Teach a GRE Math Review? Yes!, Educational Resources Information Center, ED 159 616, March, 1978, 12 pp.

This paper presented a reversed perspective on the issue of content area reading. Instead of the math teacher teaching reading, Holladay and McGinty outlined a course designed to provide a review of math topics as part of a reading and study skills laboratory. The reading specialists diagnosed students' strengths and weaknesses and planned a program of combined reading and math skills for each student. Emphasis was given to problem solving skills, which the authors contended, closely parallel reading skills. Holladay and McGinty concluded that the course demonstrated how the expertise of the reading specialist can be extended to the content area.

Woolfe, Maryanne T., "Reading Math: The Math/Reading Skills Connection," Teacher 97:76-78, January, 1980.

In order to make math more relevant to children's everyday lives, Woolfe developed a way to combine math with reading, which she felt is a high-interest subject to children. She developed a step-by-step system and guide to incorporate math skills into a variety of favorite children's stories. The article outlined the procedure in detail, and gave some specific examples from award-winning children's books.

SUMMARY

Hater, Kane, and Byrne contended that the need for acquiring the basic skills of reading mathematics has become more crucial because of increasing individual and small group instruction. It was often stated that the best place to learn how to read mathematical material is in the mathematics class (Barney, Dolgin, Earp, Florida..., Hater, Kane, and Byrne, and Krulik). These authors asserted that the mathematics teacher must become knowledgable in the skills needed to read mathematics. Earp recommended better pre-service teacher training in content reading and also in-service training programs.

Many recent authors continued to cite the 1940 study of Treacy, which determined that reading skills are directly related to mathematical ability. In a review of research studies on the relation of reading and mathematics, Aiken found a significant correlation in a representative sample of studies, and O'Mara reported that reading comprehension is important for success in math. A study conducted by Gilmary showed that reading classes have significant transfer value for arithmetic classes. Balow concluded that general reading ability and computational ability have a significant effect on problem-solving ability. Collier and Redmond, Earp, and Krulik also asserted that reading is related to success in mathematics.

According to Collier and Redmond, Smith and Kepner, Florida..., Texas..., Bye, and Schell, one of the major problems of reading mathematics is that it is an abstract subject, with a high density of concepts. For this reason, the need for a slower reading rate, and even

rereading, was stressed by many. Other difficulties associated with reading mathematics are vocabulary and symbols, comprehending mathematical material, problem solving, learning mathematical language, and reading the textbook.

Barney, Anderson, Collier and Redmond, Georgia..., Pachtman and Riley, Taschow, and Dunlap and McKnight emphasized the importance of learning the different kinds of mathematical vocabulary; most commonly identified as general, technical, and special vocabulary. Phillips and Sullivan also pointed out that the"little" words such as the, is, it, many, etc. can be critical when comprehending mathematical language. Skrypa reported a specific relationship between knowledge of mathematical terms and the ability to solve problems utilizing these terms. However, in a 1981 study by Sullivan, no significant differences in performance were found after instruction in vocabulary versus instruction in computation. The structured overview was advocated as a successful strategy for vocabulary development by Pachtman and Riley, Smith and Kepner, Thelen, and Carney. An example of this strategy can be found in appendix A.

Research by Kulm revealed that symbolism is even more difficult than vocabulary for elementary students to read. Taschow stressed that concrete experiences with operational symbols are needed, instead of memorization of definitions and principles. Wagner, however, claimed that symbols must be memorized since reading word attack skills cannot be utilized. Kulm, Anderson and Joels, Florida..., and Kane, Byrne, and Hater all noted that practice in translating symbols to words and words to symbols is necessary. This can be a difficult task since there is

not a one-to-one correspondence between math symbols and spoken words, Wagner stated. When reading 2X, X^2, and 24, the two is pronounced differently each time. Wagner further explained that the place value system causes difficulties in sequencing symbols, since a child must look all the way to the right before verbalizing a number. This need for varied eye movements when reading symbols was cited by Hater, Kane, and Byrne, Schell, Collier and Redman, Wagner, Texas..., Georgia..., Earp and Krulik. The eyes may have to move up, down, around, left, right, or diagonally. This can be difficult for children who have been taught to always read from top to bottom and left to right.

After a critical appraisal of many studies O'Mara stated that reading comprehension is important for success in math. Several comprehension skills appear to be helpful in reading mathematics, but empirical investigations on specific reading skills are scarce. In 1944 Treacy enumerated nine reading skills with a positive correlation to high math achievement. More recently Earp, Suydam and Weaver, Anderson, and Wagner identified the following comprehension skills as being necessary for mathematics: locating the main idea and relevant details, perceiving relationships, following directions, and sequencing material. Ciani, Carney, and Treacy maintained that the use of context is a valuable aid in perceiving and comprehending mathematical language. Comprehension of visual materials such as graphs, diagrams, charts, and number lines requires specific instruction since many mathematical ideas are compacted into a small amount of space. Schell recommended that the student first survey the graph for the main idea and how it is organized, then study it for detailed information. Other strategies suggested for

aiding comprehension were reading guides, study guides (such as SQ3R), graphic organizers, a visual model, and embedded organizers. Feeman stressed the importance of using a variety of strategies that will help the student to internalize the material. Bye presented an interesting perspective on students' problems with comprehending math. He proposed that many seemingly easy terms assume cognitive processes not yet available to children of that age. He indicated that broader experiences must be provided to build cognitive development and understanding of concepts.

The cloze procedure was advocated as a technique for improving comprehension of the mathematical textbook by Krulik and Lees. But the greatest acclaim for use of the cloze procedure was as a test of the comprehensibility of math material for a particular child and thus, as an aid in ability grouping. Kane, Byrne, and Hater adapted the cloze technique for use in a formula to rate the readability of instructional materials in math, since ordinary readability formulas are invalid for mathematical language. O'Mara, Smith and Kepner, and Rayburn advocated its use and effectiveness.

An accurate readability formula for mathematical material is an important tool, since research by Heddens and Smith and Elliot and Wiles indicated that the readability of mathematical textbooks was generally above the grade level for which they were intended. Elliot and Wiles pointed out that teaching and learning, dependent on a textbook that cannot be processed, comes to a halt. They denounced the attempt of some publishers to remove as much print as possible from textbooks and insisted that writers need to deal seriously with this problem.

Anderson and Joels declared that the element crucial to success in mathematics is acquisition of the appropriate language. Aiken stated that mathematics is a special, formalized language and should be taught as such. Gupta assigned parts of speech labels to mathematical symbols when he defined the structure of mathematics as a language. Cooper maintained that children should speak mathematical language before it is read or written, as with the English language. Phillips concluded that math difficulties may often be language difficulties and advocated the language experience approach to word problems. Boraks, Rezba, and Van de Walle outlined twenty elementary level activities that are designed to link language arts and mathematics. Earp and Tanner stated that students' level of language sophistication affects their success in mathematics, and said that oral discussion of mathematical ideas should be encouraged. Wagner also stated that oral interpretation of symbols, phrases, and procedures should be stressed, since memory and learning are enhanced by verbalization. Munro and McCabe each emphasized that children need the opportunity to understand and use math concepts and relations within their own natural language structure as opposed to the complex and terse language of mathematics. Syntactic structure of math materials, and word problems in specific, appears to have an effect on mathematics achievement. McCabe, Cohen and Stover, and Linville located specific syntactical patterns that can determine the difficulty of verbal word problems.

In addition to syntax, there are many variables that have been considered pertinent to problem solving achievement. But Hollander concluded that no one factor seemed to be indispensable. She stated that

intelligence, arithmetic skills, and reading skills are elements involved in successful problem solving, but conclusions from most studies were inconsistent. Balow, Linville, Stover, and Harvin and Gilchrist reported that reading ability has a significant effect on solving word problems. However, Knifong and Holtan found that poor reading ability accounted for less than 10% of the erred word problems in their study. Suydam and Weaver maintained that computational difficulties are the main deterrent, while Zweng contended that choosing the correct operation posed more problems than reading or computing.

Suydam and Weaver identified problem solving as an ultimate goal of mathematics instruction. Earle termed it the most sophisticated task in mathematics. It has also been referred to as the most dreaded task, by both students and teachers. Wagner reasoned that word problems are difficult because both reading comprehension and mathematical comprehension are required.

A variety of suggestions were offered to educators for helping children overcome the difficulties that verbal problems present. As already stated, it is imperative that the different kinds of math vocabularies be taught. Barney, Earp and Tanner, Suydam and Weaver, and Wagner emphasized verbalization of the problem. Cohen and Stover and Linville found that changing the format of word problems by drawing a diagram, reordering number sequences, or removing extraneous information can make solution easier. Davidson and Phillips advocated use of the language experience approach with word problems. Dunlap and McKnight offered an eight-step reading/thinking process; while Henrichs and Sisson suggested a directed reading lesson. Riley and Pachtman adapted a

reading guide for use with word problems that helps the student comprehend the problem on a literal, interpretive, and applied level. Maffei viewed the teaching of word problems basically as an implementation of reading study skills and advocated use of the PQ4R technique. Teachers need to experiment with these strategies to help children find the effective steps to solving word problems.

The skills presented for reading mathematics are all interrelated and dependent on each other. When students develop skills in reading math, self-confidence increases while anxiety and frustration decrease. As a result, math becomes less threatening. To help educators teach these skills, appendix A presents a list of strategies and techniques. Appendix B suggests a group of activities intended to reinforce these skills. The appendices are offered as a practical resource for interested educators.

CONCLUSIONS

Reading ability is related to success in mathematics. Research studies on the specific reading skills required to read math yielded inconsistent results, however some conclusions can be drawn.

Specific instruction of general, technical, and special vocabulary was considered imperative. Mathematical symbols pose unique difficulties because of their abstractness and density of concepts. Educators stressed that special attention should be paid to the sequencing of symbols and the directionality of eye movements. Practice in translation of words into symbols and symbols to words was emphasized.

The following reading comprehension skills were cited as requirements for successful reading of mathematics: finding the main idea and relevant details, perceiving relationships, following directions, sequencing material, using context, and extracting extraneous information. Use of a variety of techniques to aid comprehension was recommended.

Research showed that mathematical textbooks are written at a higher reading level than the grade they are intended for. Consequently, children need specific instruction in the use of the textbook. The cloze procedure, adapted to a readability formula, was advocated to determine the comprehensibility of text material.

Certain language factors affect mathematical learning. Vocabulary level, syntactic structure and format of word problems were directly related to achievement. Since it was demonstrated that verbalization of mathematical generalizations improved ability, the oral language of mathematics should be developed. Communication between teacher and student was shown to be important, and more effective if language structures from natural discourse were utilized.

Findings on problem solving were inconsistent, but many researchers found that reading ability had a significant relationship to word problem achievement. A variety of strategies were purported to be effective when teaching children to solve verbal problems.

It was stated many times that mathematical material poses unique problems for the learner because of its abstractness, conciseness, density, and highly technical vocabulary and symbolism. For these reasons, researchers and educators advised that techniques for reading mathematics can be most effectively taught by the mathematics teacher.

RECOMMENDATIONS

It is recommended that:

1. Teachers be made more aware of the unique problems that students encounter when reading mathematics through improved pre-service education and in-service training sessions.
2. As a result of increased awareness and education, teachers seek to employ a variety of strategies and activities to teach the student how to read mathematics.
3. Teachers determine the effectiveness and comprehensibility of mathematical materials being used.
4. Research be conducted to determine the relationship of specific reading skills to mathematical achievement.

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APPENDICES

APPENDIX A

Suggestions and Strategies for Teaching the Skills Needed to Read Mathematics

Vocabulary/Symbols

1. When introducing important terms, the hear-say-see-read-write sequence could be followed over a two or three day period. It is effective because all modalities are involved.

2. A new math word or symbol may be defined in terms of:
   a. a formal definition
   b. a list of attributes or characteristics
   c. a set of simulated examples
   d. a set of real-life examples
   e. a series of concept-attainment tasks

3. Make a list of unfamiliar symbols and words when previewing an assignment. Then introduce the new terms before giving the assignment.

4. Teach students how to use a glossary and a dictionary.

5. Be sure to teach the significance that little words (like articles, conjunctions, and prepositions) have in mathematics.

6. Learning a new vocabulary and symbols must be accomplished through a hierarchy of skills:
   1st word recognition
   2nd literal meaning
   3rd interpretation of meaning
   4th application of meaning

7. Teach sight word clues (shape or letters of the word).

8. Teach context clues (information obtained from print around an unfamiliar word).

9. Teach structural analysis (prefixes, suffixes, compound words, etc.).

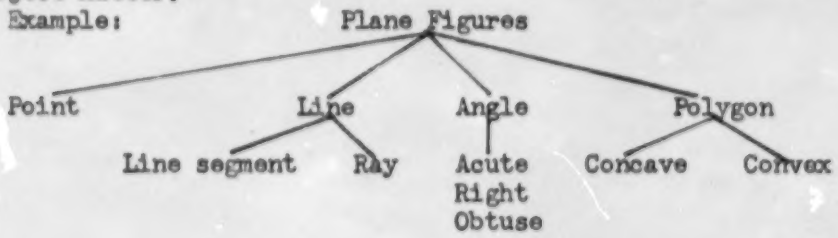
10. Teach phonic clues when needed.

Comprehension

1. Cloze procedure -- Copy a paragraph or page of a textbook. Give the first sentence in its entirety. Then omit every tenth, or fifth word. The student fills in the blanks as he reads.

2. Structured overview (also termed graphic organizer) -- This technique helps students comprehend the organization and interrelationships of terms. It is especially helpful in learning abstract subject matter.

Example:



3. Study guides -- The teacher develops a study guide on a reading passage that needs in-depth consideration. There are three stages to the guide. Items in the literal stage test the students on what the passage says. In the interpretive stage, various levels of interpretation are introduced to see what the passage is really about. It may include a pictorial level, graphic level, symbolic level, and a verbal level. In the applied stage, the guide goes beyond the passage to present new ideas based on the passage, such as story problems, theorems, and generalizations. The reader is referred to Feeman (1973) for further explanation and examples.

4. Language arts analagies -- These may be effective in helping children understand basic mathematical concepts.

Examples:

Commutative Property of Addition	John and Mary went to the store.
$4+7 = 7+4$	Mary and John went to the store.
Positional Value	Add is not the same as dad.
$384 \neq 834$	

5. When learning to read mathematical material that has diagrams, pictures, examples, etc., teach the child to use both hands. He should use one hand to keep his place in the paragraph while he points to the described visual aid with the other hand.

Language

1. Encourage oral discussion of mathematical concepts and verbal problems. This aids language development and allows the teacher to understand the thought processes of students.

2. Encourage students to restate a message in their own natural discourse to make the information more meaningful.

3. Don't talk too much. The talker is more actively involved in the learning process and in clarifying concepts. The student should be benefitting from verbalization.

4. Try tape recording parts of the textbook so children can gain experience in hearing mathematical English. This is also an effective strategy for the poor reader.

Problem Solving

1. Reading Guide -- The teacher constructs a set of statements that students respond to as they read the problem. A three-level guide is most effective: (1) literal level--the facts of the problem; (2) interpretive level--mathematical ideas underlying the problem; (3) applied level--numerical depictions of the problem. Refer to Riley and Pachtman (1978) for an example of a word problem reading guide.

2. Directed Reading Method -- A step-by-step approach to solving word problems.
   - step 1: Read through the problem quickly for a general idea of the problem as a whole.
   - step 2: Re-examine the problem slowly to determine what is asked.
   - step 3: Note the exact information given.
   - step 4: Analyze the relationship of given information (step 3) to what you are asked to find (step 2).
   - step 5: Translate the relationship to mathematical terms.
   - step 6: Perform the computation.
   - step 7: Examine the solution to see if it is sensible. Label it.

3. Give students practice in estimation. It is an essential skill of reading comprehension whenever numbers are involved.

4. Urge children to have instant command of the basic number facts. They are an integral part of the thought process, just as sight words are to reading. By the end of the fourth grade they should be mastered.

5. Write the word problem across the board in one long line. Below this, substitute the mathematical symbols for the words. This translation device can be effective in showing how the verbal statement can be written in a mathematical statement.

6. Encourage students to read word problems orally. Sometimes poor readers can solve the problem if they hear it.

7. Simplify word problems for handicapped readers. Make the general vocabulary easier, simplify sentence structure, and shorten the sentences. Ditto the simplified problems and file away for use year after year.

8. Try language experience word problems so students can see that verbal problems can relate to daily living.

9. Have students mark irrelevant details as the problem is read and worked through. This will help to prevent students from using incorrect data in their computation.

This is a compilation of suggestions from the authors of the sources annotated.

APPENDIX B

Games and Activities to Reinforce the Skills Needed to Read Mathematics

Vocabulary/Symbols

1. "Concentration" -- Make a set of cards on which are written symbols or the words for that symbol (=, equals, <, is less than, X, multiply). The cards are all turned upside down and the players take turns trying to match pairs.

2. Have each student make his own glossary or math dictionary. The mathematical definition for each term should be stated in the student's own words, and also the everyday definition when applicable. The word should be used in a sentence.

3. Develop acrostic puzzles, crossword puzzles, hidden word puzzles, etc. to give students practice in learning new vocabulary.

4. "Math Symbol Bingo" -- Make bingo cards with any group of symbols you are trying to reinforce.

   Example:

$\cap$	$>$	$\leq$
$\emptyset$	free	$\div$
X	$\neq$	=

5. "Shazam!" -- Cut about 50 cards and lable them with a math symbol, word, or phrase. Make 5 cards labeled "Shazam!" Three to six children sit in a circle and take turns pulling a card from a can. If the card can be read, the child keeps it--if not, it's returned to the can. If a "Shazam!" card is selected, the player loses all of his cards. The first player to get five cards wins.

6. Make puzzle pieces that fit together when symbols and words are correctly paired.

   Example:

7. Have students complete matching exercises such as the following:

___ 1. is equal to	a. >
___ 2. is greater than	b. $\cup$
___ 3. intersection	c. =
___ 4. empty set	d. $\cap$
___ 5. union	e. $\emptyset$

8. For words having both general and specialized meanings, have the student write two sentences for each word, depicting both meanings.
   Example: Joan covered her legs with a blanket.
   The legs of an isosceles triangle have equal measures.

9. Have students write mathematical sentences in as many different forms as possible. (Besides reinforcing symbols and vocabulary, this helps young students understand the flexibility in sentence order.)
   Example:
   398 = 300 + 90 + 8
   398 = (3x100) + (9x10) + (8x1)
   three hundred ninety-eight equals 300 + 90 + 8
   398 = three hundred plus ninety plus eight

10. "Bean Bag" -- Divide a large piece of posterboard into 24 or 30 squares and put a symbol or term in each box. Laminate the board. Child on one team tosses a bean bag onto the board. If he can tell about the symbol or term, his team gets a point. The team with the most points wins.

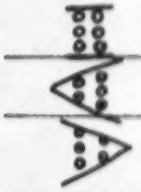
11. Using cards with mathematical symbols or phrases on them, have a student read them orally while the class writes the expression.

12. "Find Your Partner" -- To help children learn abbreviations, divide the class into two groups. Give one group cards with mathematical words. Give the other group cards with abbreviations for the same words. At the signal, the groups "scramble" to find their partner (matching card).

13. To help children be able to express an idea in objects, pictures, words, and symbols, try exercises like the following:
   Fill in the empty spaces:

Symbol	Word	Picture
	three	
1/2		
	angle	
$\overleftrightarrow{AB}$		

14. Work with pictorial development of meanings as well as words:



$3 = 3$	"is equal to"
$2 < 3$	"is less than"
$3 > 2$	"is greater than"

15. To help students become aware of multiple meanings for some symbols and words, make charts that show the different uses:
    a. discuss the meaning of "-" in each of the following:

$\overline{AB}$ $\quad 8 - 2 \quad -4 \quad 3.\overline{3}$

    b. general "point":

    technical "point": 34.5
    $1.67

Comprehension

1. Mimeograph pages from the textbook and mount each on posterboard. Cut the page apart into a jigsaw puzzle, which the student has to reassemble. This encourages slow and careful reading to find the full meaning of sentences and the page.

2. Write some word problems in the form of a newspaper article. Ask students to write a headline for the article which summarizes the main idea in question form.

3. Help students concentrate on key words with the following activity: Directions: Write the correct symbol for the operation suggested by each expression.

1. the sum of +
2. take away ___
3. the divisor ___
4. altogether ___
5. the product ___
6. more than ___

4. To help students see patterns and relationships, give them exercises like the following:

   Directions: For each group of terms, draw a circle around the one which does not belong with the others. Explain your answers.

1.	apple	pear	chair	banana
2.	add	subtract	multiply	vanish
3.	factor	sum	multiply	product
4.	five	add	four	three
5.	2	4	5	6
6.	13	15	11	7
7.	1	3	6	9
8.	4	8	14	20

5. Have the class read a page or passage in their math book and then answer questions like:
   a. What is the main idea?
   b. What are the supporting details?
   c. How are the main ideas and details related?

6. The ability to read mathematical sentences is needed to comprehend mathematics. Have students answer specific questions about mathematical sentences.

   Example: $6 + 7 = 13$

   1. What is the operation?
   2. What does the "1" tell you? the "3"?
   3. Rewrite the sentence using words.
   4. Is it a true sentence?

Problem Solving

1. Select several problems from the textbook. Ask the students to rewrite them using the same information and data, but different context.

   Example:

Tom has six bags with four bananas in each bag. How many bananas does he have altogether?	Jeff has six cars. Each car has four wheels. How many wheels are there altogether?

2. Supply the students with problems that contain no numbers. In small groups, let the students decide how to solve the problem and what additional information is needed.

   Example: Angie wants to buy a package of gum and a candy bar. How can she decide if she has enough money?

4. Game: "Have A Problem?" -- On the front of a 3x5 card, write the half of a story problem that gives the information (for example, "If John had 72 apples and received 19 more..."). On the back of the card, write the question and the process needed to solve it ("How many apples did he have altogether? -- addition). To play the game, place the cards in a pile, information side up. Each player draws a card and tries to guess the question and process. If he was correct he keeps the card; if incorrect, he discards it. The player with the most cards, when they're all used, wins.

5. Extraneous Information -- Write out some word problems that have extraneous information. Mount them on tagboard and laminate. Have children use a felt-tip pen to cross out all words or numbers that are not necessary to solve the problem.

   Example:

   Kevin worked two hours mowing a lawn that was 125 feet long and 75 feet wide. He was paid $1.25 an hour. How much did he earn?

6. Without Words -- Write out some math number and symbol sentences. Have children make up a story problem to fit the number/symbol sentence.

   Example: $\frac{1}{2} \times 36 = n$

   There are 36 students in Lisa's gym class. Half of them were on team A, and half were on team B. How many children were on each team?

Using the Mathematical Textbook

1. To help students become familiar with parts of the text, such as table of contents, glossary, index, appendices, etc., have a scavenger hunt. Make a list of locational type questions about the text. Divide the class into two teams. The team that finds the answer first to each question, as it is asked, gets a point.

2. Help students adjust their reading rate to a careful, slower pace. Read a light, humorous story to the class at a comfortable, but rapid rate. Then read their math book to them at the same pace. The need to adjust the rate of reading becomes obvious.

3. Have students scan a page in their mathematics book and summarize what they learned. Next, have them read it carefully and again summarize learning. Compare the two experiences.

This is a compilation of suggestions from the authors of the sources annotated.